Newton's second law is a fascinating piece of physics because it clearly is the basis of one of the most successful pieces of theory ever developed, namely Newtonian mechanics, yet, if we believe a lot of what has been written about it, it appears to be a tautology. Well it isn't a tautology, so a lot of what has been written about it is obviously wrong.

Very briefly, in the non-relativistic limit, Newton's second law is $F = ma$ where $F$ is the resultant force acting on a body of mass $m$ and $a$ is the consequent acceleration of the body, and we are told with monotonous regularity that it applies only to measurements made relative to an inertial reference frame. Well, that is where the trouble starts. How do you identify an inertial reference frame, i.e. one which is not accelerating? Not accelerating relative to what? Not accelerating relative to the fixed stars, we are often told. Why the fixed stars? What have they got to do with it? Not accelerating according to an accelerometer fixed to the reference frame, we are often told. Well no one has ever succeeded in designing a suitable accelerometer and, according to Einstein's principle of equivalence, no one ever will.

Consider a simple accelerometer as illustrated. If this is given
an acceleration \( a \) to the right, the pointer will move along the scale to the left until the resultant force \( F \) which the springs exert on the mass \( m \) is given by \( F = ma \). As long as the pointer indicates that \( F = 0 \), the claim is that the frame is not accelerating. Well rotate it so that is is vertical instead of horizontal. It will settle at a reading of \( F = mg \). Are we accelerating vertically with acceleration \( g \) all the time? Of course not! Put it a capsule in free fall. It will register zero. Is an object in free fall not accelerating? Of course it is accelerating! Well relative to the earth and relative to the fixed stars it is, anyway. It looks as though the accelerometer idea doesn't have much future.

What about the fixed stars then? Can we find a reference frame which is not accelerating relative to the fixed stars? No we can't. What about the surface of the earth? No good: the earth is spinning about its axis and orbiting the sun, and this accelerated motion relative to the fixed stars is readily observable from observations made on the stars. Could you perhaps find such a reference frame out in space? I don't like your chances, and I'm not specially interested if you can. It so happens that I do my physics here on earth, and it so happens that Newton discovered his law here on earth.

We all use \( F = ma \) every day in a reference frame which is clearly accelerating. How do we get away with it? Is it only approximately true on the surface of the earth, even in the non-relativistic limit? There is no evidence to support this view. Do we have to subtract off the effects of the earth's rotational and orbital motion (calculated by means of Newton's law!) before we can use Newton's law? No we don't! Then what is all this about Newton's law being true only in an inertial reference frame?

The trouble arises when we discuss the \( F \) of \( F = mg \). The \( F \) of
\( F = ma \) is the resultant of the forces acting on the body, and here on earth we are well aware that one of the forces contributing to this resultant is the gravitational or weight, force. The key question with regard to the weight force is "how do you measure it?" Can you, in principle, measure it with a spring balance? Can you, in principle, measure it by making observations on freely falling bodies in vacuum? The answer to both questions is, of course, yes. But if we are trying to get to the bottom of Newton's second law we had better leave out the freely falling body method because it will involve an assumption that Newton's law is correct in our laboratory reference frame. Actually a spring balance tells us two things. It tells us what direction is vertical and it tells us the magnitude of the weight force.

Let us now consider a mass hanging from a spring balance, and let it all be at rest relative to me, but accelerating from left to right with acceleration a relative to you (who are comfortably at rest on mother earth). According to you, (a),

![Diagram](image1.png)

\( \dot{F} = ma \) and \( F = T + W \) where \( T \) is the reading of the spring balance, \( W \) is the weight force, and \( OZ \) is the vertical. According to me, (b), \( F = ma \) but \( \ddot{F} = T + W' = 0 \) and \( Om \) is the vertical. We agree with each other on every issue except the magnitude and direction of the weight force.

According to you \( T = ma - W \)

According to me \( T = -W' \).
The magnitude of $\mathbf{T}$ is not in dispute; it is the spring balance reading, but our values for the weight force differ by $m\mathbf{g}$.

$$W' = W - m\mathbf{g}$$  \hspace{1cm} (1)

Because I am accelerating relative to you I get a different value for the weight force and a different direction for the vertical. Who is right? We'll save the answer until later.

Consider now that our mass plus spring balance is rotating as a conical pendulum with angular velocity $\omega$, as seen by you, but that I am very cleverly staying alongside $m$ all the time and remaining at rest relative to it. According to you, (a), $F = m\mathbf{g} = T + W = F_c$ towards $C$, and $OZ$ is the vertical. According to me, (b), $F = T + W' = 0$ and $Om$ is the vertical. Again we have different values for the weight force and the vertical.

$$W' = W - m\mathbf{g} = W - F_c = W + (-F_c)$$  \hspace{1cm} (2)

Who is right?

Compare equations (1) and (2). In both cases my estimate of the weight force on $m$ differs from yours by the addition of an amount $-m\mathbf{g}$. 
There is a force acting on \( m \), which I identify as part of the weight force \( W' \), which is proportional to the mass of the body and to my acceleration relative to you. Thus the weight of the body is given by you as \( W = mg \), and by me as \( W' = m(g - a) \) where \( a \) is my acceleration relative to you.

How can the weight force on a body depend on the acceleration of the observer measuring it? What right have I to make an arbitrary change in \( g \) by calmly adding \(-a\) to it, just so that I can claim the right to use \( F = ma \)? Well first let me ask what \( g \) is. Is it \( \frac{W}{m} \)? Is it the acceleration of a freely falling body? Is it \( G \frac{M}{R^2} \), where \( M \) and \( R \) are the mass and radius of the earth? Or is it simply \( \frac{T^2}{m} \), where \( T \) is the reading of your spring balance when you weigh the body? If you settle for \( G \frac{M}{R^2} \) you are in real trouble because there is no way you can measure it. The other three are all the same as each other and may be measured accurately with a pendulum of equivalent length \( \ell \).

\[
g = \frac{4\pi^2}{\tau^2} \ell
\]

where \( \tau \) is the period of the pendulum. But why your balance, and why your pendulum? What's wrong with mine? Don't tell me mine is accelerating or rotating or whatever else, because so is yours, and in fact this is the very reason that \( G \frac{M}{R^2} \) differs from the other definitions of \( g \). If you measure \( g \) by whatever method you fancy, you will get \( G \frac{M}{R^2} \) towards the centre of the earth plus \( \omega^2 R \cos \theta \) away from the centre of your parallel of latitude, where \( \theta \) is your latitude.
Thus the weight force on mass $m$, as measured by you, will differ from $G \frac{Mm}{R^2}$ by the addition of a centrifugal force $m \omega^2 R \cos \theta$ exactly as was my weight force when we were talking about the conical pendulum. Notice that you cannot make a physical measurement of $G \frac{Mm}{R^2}$ or $m \omega^2 R \cos \theta$ separately. You can measure only their resultant, and it is the direction of the resultant which you call vertical. Having measured their resultant you are then free to play around theoretically and extract these two components if it amuses you, but it is the resultant to which the body responds and the resultant to which any measuring device responds. My $W'$ is therefore no less fundamental than your $W$, and far from having to justify my casual addition of an acceleration $-\mathbf{a}$ to $\mathbf{g}$ so that I can claim the right to use $\mathbf{F} = m \mathbf{a}$, not only I, but also you, have no choice in the matter. We both do it and there is no way of avoiding doing it.

If there is no way of sorting out $\mathbf{g}$ from $\mathbf{a}$ in $(\mathbf{g} - \mathbf{a})$ can we perhaps have a second try at locating an inertial reference frame? Suppose, instead of trying to find a frame with $\mathbf{a} = 0$, we seek one with $(\mathbf{g} - \mathbf{a}) = 0$. This corresponds to weightlessness and therefore to an observer in free fall. The accelerometer we discussed earlier is a
valid tool for identifying such a frame, and the ideal inertial reference frame becomes simply one in which no gravitational field is detectable.

There is still the nagging question of how acceleration of the reference frame can change what we instinctively think of as the gravitational force. The clue lies in our very first suggestion for a definition of an inertial reference frame, one which is not accelerating relative to the fixed stars. The $\omega$ in the term $m \omega^2 R \cos \theta$ for the centrifugal force component of any weight measurement made on the surface of the earth is the angular velocity of the earth about its axis relative to the fixed stars. This is not a special case, the weight force is always made up of two components, one due to gravitational attraction of massive bodies and the other, called an inertial force, due to acceleration relative to the fixed stars. Physically they are indistinguishable and inseparable. Their indistinguishability is the subject of Einstein's principle of equivalence and their inseparability is the empirical evidence on which this principle is based. There is a widespread custom of calling the gravitational force a real force and the inertial force a fictitious force. The basis of this distinction is the fact that we have an established theory for the gravitational force but, as yet, not for the inertial force. The rationale for this line of thinking completely escapes me. A force we cannot measure is called real and one we can measure has a component called fictitious! The reality of a force is manifested in its influence on the behaviour of masses, not in the existence or non-existence of a theoretical understanding of its origin. If a body is set spinning about an axis with sufficiently high angular velocity it will be disrupted. If the body were your body you would feel the centrifugal force tearing at you radially outwards and if this force exceeded the maximum possible intermolecular force between your molecules you would be ripped to pieces.
It would take much more than the protestations of a pious believer in the sanctity of inertial reference frames to convince you that this force which was about to dismember you was fictitious! Certainly, the inertial observer could also explain your pending destruction: he would say that your maximum intermolecular force was not strong enough to provide the central resultant force required to hold all your parts in circular motions. His explanation is perfectly good, *every bit as good as yours, but no better than yours*, and not as simple as yours. The answer to my earlier question "who is right?" when two observers gave different values for weight and different directions for vertical is then that both are right.

There are many other examples where the simplest explanation is that of the observer in the rotating reference frame, e.g. the raising of tides, the operation of a centrifuge, and the parking of a satellite above a fixed point on the earth's equator, but the only other one I shall discuss concerns the emission and absorption of γ-rays by the atomic nucleus iron-57. When the first excited state of iron-57 decays 14.4125 keV of energy is released. If the atom was free, most of this energy would appear as a γ-ray, but a small part would appear as recoil of the iron-57 nucleus. The γ-ray would therefore have an energy slightly less than 14.4125 keV. Similarly if a γ-ray is to be absorbed by an iron-57 nucleus in its ground state and raise it to its first excited state the incident γ-ray must be of energy slightly greater than 14.4125 keV because the absorbing nucleus would also recoil. However, if both emitting and absorbing nuclei are bound into iron crystals, the recoil is negligible and the γ-rays emitted by one nucleus have exactly the right energy to be absorbed in another, provided the emitter and absorber are in a horizontal plane. Place the emitter a height $H$ (a metre or so) above the absorber and the γ-rays are not absorbed because of the energy they
have gained in falling in the earth's gravitational field. Now make the
absorber move vertically down, away from the emitter, and at some speed v
the Doppler shift exactly compensates for the effect of the gravitational
field, and the γ-rays are absorbed again. Now put the emitter at the
centre of a rotating turntable and put the absorber at some distance R
from the axis. The absorber experiences a centrifugal force,
indistinguishable from a gravitational force, and the γ-rays are not
absorbed. Give the absorber a velocity v radially outwards and the
Doppler effect again does the trick and the γ-rays are absorbed. Provided
the difference of gravitational potential \( \int_0^R g \rho \, dr = gh \) in the first instance
is equal to the equivalent expression \( \int_0^R \omega^2 r \, dr = \frac{1}{2} \omega^2 R^2 \) in the second
instance, the same velocity v is required in both cases! I frankly don't
know how the inertial observer explains this. I don't doubt that he
can, but his explanation will be very involved.

But I digress. We were talking about the importance of the fixed
stars. If acceleration relative to the fixed stars is the root cause of
the appearance of inertial forces, then the stars must somehow be
responsible. This is known as Mach's principle. If inertial forces are
indistinguishable from gravitational forces, presumably they are
 gravitational forces, gravitational forces resulting from our acceleration
relative to the fixed stars. At any rate this is the view that was taken
by Einstein in his principle of equivalence. In short, just as the
electromagnetic field is made up of three parts, the electrostatic (or
Coulomb) field, the magnetic (or velocity dependent) field, and the
radiation (or acceleration dependent) field, so we claim that the
gravitational field has its static (or Newtonian) part and its inertial
(or acceleration dependent) part.

What about a velocity dependent part? Well yes, there is one of
them too. It is called the Coriolis force. For reasons associated
with the fact that there is only one sign of mass (whereas there are two signs of electric charge) the Coriolis force is observable only in a rotating reference frame, and because it is a frame dependent force it also is called an inertial force. We shall consider two simple cases, those of a particle moving radially and a particle moving circumferentially relative to a rotating turntable.

Suppose a particle moves at constant speed \( v \) along a radial groove in a turntable which rotates with angular velocity \( \omega \). When it is distance \( r \) from the axis of rotation its angular momentum, according to an inertial observer watching from beside the turntable, is given by

\[
L = mvr^2
\]

Let us now write Newton's second law in the form

\[
F = \frac{dp}{dt}
\]

where \( p \) is the momentum of the body.

For angular motion this becomes

\[
\tau = \frac{dL}{dt}
\]

where \( L \) is the angular momentum and \( \tau \) is the torque causing the change of angular momentum. Thus in our case, the mass \( m \) must experience a tangential force \( F \) given by

\[
rF = \frac{d}{dt}(mvr^2) = 2mvr \frac{dr}{dt} = 2 m\omega rv,
\]

whence

\[
F = 2 m\omega v.
\]
That is, the wall of the groove exerts a force equal to $2mv\omega$ on $m$, in a
direction perpendicular to that of $v$ and perpendicular to the axis of
rotation (which is the direction of $\omega$ if it is written as a vector).
To an observer at rest on the turntable $m$ is simply moving at constant
speed along a radius. It therefore experiences no resultant force. Yet
the wall of the groove is pushing on $m$ with a force $F = 2mv\omega$. Hence
there must be some force ($= 2mv\omega$) acting on $m$ in the opposite direction
to $F$. This is the Coriolis force, an inertial force, a weight force.

Now consider $m$ to be moving on the end of a rope at constant speed $v$
relative to the turntable, in a circle of radius $r$. Our inertial
observer sees $m$ moving with speed $v + \omega r$, with the tension of the rope
providing the central force. Hence the tension in the rope is given
by

$$T = \frac{m(v + \omega r)^2}{r} = \frac{mv^2}{r} + m\omega^2 r + 2mv\omega.$$  

But the observer on the turntable requires a central force of only $\frac{mv^2}{r}$
to account for the circular motion he observes. Yet the tension in the
rope is $T$. Hence there must be some force ($= m\omega^2 r + 2mv\omega$) acting on $m$
radially outwards; $m\omega^2 r$ is the centrifugal force in the rotating
reference frame, and $2mv\omega$ is again the Coriolis force. Hence the
Coriolis force on a body is proportional to the mass $m$ (as any
gravitational force should be) is proportional to the velocity of the
body and to the angular velocity $\omega$ of the turntable, and is perpendicular
to \(v\) and to \(\omega\). Compare this with the magnetic force on a moving charge \(q\) in a magnetic field \(B\). \(F = qvB\) and \(F\) is perpendicular to both \(v\) and \(B\).

[In vector notation \(F = \vec{q} \times \vec{B}\) (magnetic) and \(F = 2m\vec{v} \times \vec{\omega}\) (Coriolis)]

Where do we experience Coriolis forces? Try walking around on a merry-go-round and you will experience it at first hand. Have a ride on the Rotor at Luna Park and you will experience the centrifugal force holding you "down" against the vertical wall! and if you move your arms they will be deflected sideways. When a low pressure centre develops in the atmosphere the surrounding air immediately moves towards it, but thanks to the Coriolis force due to the rotation of the earth, the air is deflected sideways and \(\text{spirals in to the centre (clockwise in the southern hemisphere and anti-clockwise in the northern hemisphere).}\) All our weather patterns are dominated by Coriolis forces, and heaven help the meteorologist who tries to describe them from the point of view of an intential observer!

Inertial forces, which term embraces all those components of the weight force which depend on the acceleration of the reference frame relative to the fixed stars, are part of our everyday existence. We feel them. We can measure them. The fact that we cannot properly explain them does not make them fictitious or artificial, it simply demonstrates one of the limitations of current physical theory. They are weight forces and are just as real as any other weight forces.

Let me conclude by giving you an example of a force which really is non-existent. It is called centripetal force. It is just a name, and has no physical reality at all, in \textit{any} reference frame!