LECTURE 2

Light Propagation

How is it influenced by matter?

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1. Introduction

In this lecture we shall attempt to understand the observed properties of light propagation in matter in terms of several simple concepts:

These are:

1. The Maxwell picture of an electromagnetic wave in vacuum.
2. The Hertz theory of radiation by vibrating charges.
3. The atomic picture of matter in which matter is made up of charged particles.
4. The usual kinematical ideas on wave propagation.

The first major theoretical synthesis along these lines was carried out by H.A. Lorentz in 1906, and he was extraordinarily successful, inspite of the lack of the quantum theory of matter and electromagnetic radiation, and inspite of the absence of the Rutherford nuclear atom.

This situation still holds, for Dirac's 1926 quantum theory of the electromagnetic field led to few changes, if any, when applied to ordinary optics. Further, the quantum theory of matter developed by Heisenberg, Schroedinger and others in the mid-1920's lead mainly to quantitative differences, when applied to optics in the visible region. All the phenomena and their qualitative explanations remained intact. (The Compton and photoelectric effects at higher energies are notable exceptions).

Thus, unashamedly, our considerations will be made largely in the spirit of classical physics.

One final comment before I begin; the nature of an explanation in this area of physics has changed dramatically. Before the atomic theory, light was refracted because it moved slower in material media. After atomic theory light was refracted by virtue of its interaction with the atoms of the optical media. No less than the full dynamical details will now satisfy the physicist.

2. Electromagnetic Waves in Vacuum

Since the time of Maxwell, light has been regarded as an electromagnetic wave. In this view a sinusoidal wave train consists of a mutually orthogonal electric and magnetic field both at right angles to the direction of propagation. This situation is illustrated in figure 1.

![Diagram of Maxwell picture of a plane polarised sinusoidal electromagnetic wave.](image-url)
In these lectures the plane of the electric field will be referred to as the plane of polarisation. The electric field \( E \), the magnetic field \( B \) and the direction of propagation, form an orthogonal right-handed set of directions. In the visible region the wavelengths range from about 4000 Å (Angstrom)\(^{8}\) in the extreme violet to 7200 Å in the deep red. By comparison, atoms have radii of order 2 Å. In solids and liquids the atoms are a few Angstroms apart, whereas in a gas at a temperature of 0°C and a pressure of 1 atmosphere the atoms have a separation of about 33 Å.

We see therefore that a single wavelength of light envelopes hundreds, if not thousands, of atoms in its passage through matter.

3. Scattering of light by bound charges

Light scattering occurs as a two-step process. Firstly, the incident light wave sets the charges of the system into forced oscillations at the frequency of the light wave. Secondly, the oscillating charges, by virtue of their accelerations, radiate an electromagnetic field. It is this additional radiated field that is identified with the scattered light wave.

Consider a sinusoidal light wave, polarised in the \( x \)-direction, incident on an elastically bound particle of mass \( m \) and charge \( q \).

Newton's equation of motion reads:

\[
\text{(mass \times acceleration) = (elastic restoring force) + (sinusoidal electric force)}
\]

\[
m \frac{d^2x}{dt^2} = -m\omega_0^2 x + qE_0 \cos \omega t
\]

(1)

In this equation \( \omega_0 = 2\pi f_0 \) and \( \omega = 2\pi f \), where \( f_0 \) and \( f \) are the frequencies of free oscillation of the charge and of the electromagnetic wave, respectively. \( E_0 \) is the amplitude of the electric field of the incident light wave.

Equation (1) has the solution

\[
x = \frac{q E_0 \cos \omega t}{m(\omega_0^2 - \omega^2)}
\]

(2)

We note in passing a change in phase associated with \((\omega_0^2 - \omega^2)\), which depends on whether \(\omega < \omega_0\) or \(\omega > \omega_0\). (This change of phase depending on whether the driving frequency is above or below the natural frequency may be illustrated by studying the oscillations of a ball on an elastic string).

The electric field at time \( t \) radiated by an accelerated charge is given by \( E_\theta \), where

\[
E_\theta = E_\theta(r, \theta, t) = \frac{-qa}{4\pi\epsilon_0 c^2} \cdot \sin^2 \frac{\omega t}{r}
\]

(3)

In this equation \( c \) is the velocity of light and \( \epsilon_0 \) the permittivity of the vacuum. The distance from the charge to the observer is \( r \), in a direction \( \theta \) relative to the line of acceleration. The acceleration \( a \) has to be evaluated at the time the radiation was emitted, i.e. at time \( t = 0 \). The geometry is shown in figure 2.

![Figure 2. Geometrical relationship between the accelerated charge and the radiated field](image)

Please purge your minds of Angstroms if you are young enough, and replace them everywhere by nanometer - the "nm" unit. 10Å = 1 nm.
The polarisation vector of the radiated wave and the acceleration vector lie in the same plane.

The intensity pattern is shown in figure 3. It will be noted that there is no radiation along the line of acceleration, and that the intensity is greatest in the broadside direction. We may readily observe this angular intensity variation in the scattering of polarised light by a colloidal solution. The scattering is strongest in the plane of polarisation, and is absent in the plane perpendicular to the polarisation.

![Figure 3: The intensity I of radiation emitted in direction θ, relative to the acceleration. I = sin^2θ](image)

Using equation (2) to calculate the acceleration of the particle, we may now evaluate equation (3) for the scattered field.

\[
E_s = \frac{q^2/m}{4\pi\varepsilon_0 c^2} \frac{r^2}{f_0^2 - f^2} \sin^2 \phi \cdot E_0 \cos 2\pi f \left(t - \frac{r}{c}\right)
\]  

(4)

It will be observed that radiation is strong when the light frequency \( f \) is near the natural frequency \( f_0 \). Further there is a change of phase depending on whether \( f < f_0 \) or \( f > f_0 \) by virtue of the factor \( (f_0^2 - f^2) \).

The magnitude and phase of \( E_s \) are shown as a function of \( f \) in figures 4a and 4b.

![Figures 4a and 4b: The amplitude and phase of scattered radiation.](image)

We note that equations (2) and (4) predict an infinite oscillation when \( f = f_0 \), i.e. when the incident light frequency equals the natural frequency of vibration. In fact such infinite oscillations do not occur, because for large amplitudes of vibration dissipative processes come into play which damp down the vibrations. ("Nature abhors an infinity").

The true scattered fields and phase is shown in figures 5a and 5b.

![Figures 5a and 5b: The amplitude and phase of the scattered radiation allowing for damping processes.](image)
4. Refraction

We know that refraction, the bending of light rays at an interface, occurs because the velocity of the waves is different on both sides of the face. We turn our attention here to the question: what controls the velocity of light in an optical medium?

When a plane light wave falls on an atom an outgoing wave is generated as shown in figure 6.

![Figure 6. Light waves scattering from a single scattering centre.](image)

It will be observed that only in the forward direction does the plane wave and scattered wave move with the same velocity.

We ask - what will the total scattered field look like from a thin layer containing many atoms? We are here thinking of a layer, thin compared with the wavelength of light. Consider figure 7:

![Figure 7a shows the configuration of plane wave falling on a thin slab of atoms. In figure 7b, the slab of atoms is shown faceted. The atoms in the annular regions all contribute equal amounts to the field at point p, but with a different phase in each region.](image)

We ask - what is the total scattered field at point p from the slab? We have to add up the contributions from successive annular regions in the slab, as is shown in fig. 7b.

Let \( t \) be the thickness of the slab, and \( N \) the number of atoms per unit volume, and \( dA \) a patch of area. The total scattered field \( E_{ts} \) at \( P \) is given by

\[
E_{ts} = \int E_{s} \cdot N \cdot dA
\]

(5)

In this expression \( E_{s} \) is the field at \( P \) due to a single atom in the patch, and \( N \cdot dA \) is the number of atoms in the patch \( dA \).

Putting in \( E_{s} \) from eq. 4 and performing the integration (which is tricky) we find for the total scattered field

\[
E_{ts} = \frac{q/m}{4\pi c} \cdot \frac{f}{r_{o}^{2}-r^2} \cdot E_{o} \cdot \sin 2\pi f(t-z/c)
\]

\[= E_{tso} \cdot \sin 2\pi f(t-z/c)\]

(6)
where $E_{t0}$ is the amplitude of the total scattered wave. We note that the total scattered wave is a plane wave, as was the incident wave, but that it is $\pi$ (90°) out of phase with it. (Note that the function is Sine and not Cosine). This phase shift comes from the fact, that the wavelets scattered by the atoms to the point P have on the average come by a longer path than the direct path. From a thin section of material the scattered wave is weak compared with the incident wave.

We note in passing that a similar scattered wave appears on the other side of the slab, travelling the other way, of this wave more later. The total electric field, $E_t$, on the far side of the slab is given as the sum of the incident and total scattered fields.

These fields are illustrated in figure 8 for the time $t=0$.

\[
E_t = E_0 \cos 2\pi f(t-z/c) + E_{t0} \sin 2\pi f(t-z/c) \tag{7}
\]

![Incident and Scattered Wave](image)

**Figure 8a.** The incident and scattered wave. **Figure 8b.** The total field. For comparison the field that would be present in the absence of the slab is shown dashed.

It will be observed that the effect of the layer of atoms is to slide the total field in closer to itself by a small distance $d$, shown in figure 8b. Except for this sliding along the total field is similar to the incident field.

A small amount of trigonometry applied to equations 6 and 7 yields

\[
d = \frac{a^2}{8\pi^2} \frac{N}{F^2} \cdot \tau \tag{8}
\]

where $K$ is a constant. i.e. The amount of shifting is proportional to the slab thickness.
Consider now a full block of material. If we regard the block of material as made up of thin parallel slabs, each slab has as incident field the total field coming from the previous slab. Each slab "pulls" the wave toward itself, thereby shortening the wavelength in the material.

Let $\lambda$ be the wavelength of light in the material, and $\lambda_0$ the wavelength of light of the same frequency in vacuum.

In travelling a distance $\lambda$ in the material the wave has been shortened by an amount $K\lambda$ (equation 8) from its vacuum value $\lambda_0$. Thus the vacuum wavelength is given by

$$\lambda_0 = \lambda + K\lambda$$  \hspace{1cm} (9)

Now the refractive index $n$ is given by

$$n = \frac{\text{velocity in vacuum}}{\text{velocity in material}} = \frac{f\lambda_0}{f\lambda} = \frac{\lambda_0}{\lambda} = 1 + K$$

Thus we find, at last, from equation 8 the celebrated equation for the refractive index

$$n - 1 = \frac{q^2/m}{8\pi^2\epsilon_0} \cdot \frac{N}{f^2 + f_0^2} \cdot N$$  \hspace{1cm} (10)

This expression is shown graphically in figure 9, with allowance made for damping.

![Figure 9](image)

**Figure 9.** Refractive index $n$ as a function of frequency

Expressions of type 10 are known to represent the refractive index rather well over a large frequency range. The experiments of Bloch have verified the form in detail for sodium vapour.

Let us suppose $f < f_0$ so that there is no phase delay on scattering. The wave amplitude at each point is the result of scatterings from the previous sheets of atoms, and this wave becomes the incident wave for later scatterings.

If we could imagine that we could "tag" a small part of the wave energy, the path it might take is shown in figure 10.
The velocity from AB, BC, CD, etc. is the vacuum velocity. The wavelength in the material appears shorter, i.e. refractive index increased, because the vacuum wavelength has been "crumpled up" by successive scatterings.

If $f > f_0$ there is a half cycle phase delay, which for sinusoidal waves can also be regarded as a phase advance. This may make the wave look as if it had overcome the effects of delays due to the path complexities of multiple scattering and had "sped up". The refractive index thus dips below unity. Detailed calculations by Brillouin and Sommerfeld have shown that at all times light pulses actually travel more slowly than waves in a vacuum. Relativity is not violated.

5. Criticisms of the Theory

There are several main criticisms of the theory:

(i) For dense media the intense local electrostatic field of the oscillating charges exerts a strong influence on the neighbouring atoms. Clausius and Moessbauer showed that the replacement in equation 10

$$n - 1 = \frac{(n^2 - 1)}{n^2 + 2} \cdot \frac{3}{2}$$

was the necessary modification required.

(ii) Atomic and molecular systems have more than one resonant frequency in general. This is accommodated in equation 10 by replacing the left hand side by a sum over the various natural frequencies. Altogether, we find

$$\frac{n^2 - 1}{n^2 + 2} \cdot \frac{3}{2} = \frac{\sigma^0 N}{8\pi^2 e_0 m} \sum_{i} \frac{F_i}{F_i^2 - f^2}$$

where $e$ is the charge of the electron and $m$ its mass. $F_i$ is the effective number of electrons participating in the oscillation at frequency $f_i$. \( \sum_{i} F_i = Z \), where $Z$ is the total number of electrons in the atom or molecule.

The frequencies $f_i$ being the natural frequencies of oscillation of the atomic structure correspond to the spectral line frequencies emitted, (in transitions of the ground state). Thus sodium vapour, which has the two well known closely spaced yellow spectral lines was found by Wood to have a
refractive index which underwent marked fluctuations in accordance with equation 12 at these two frequencies. This is illustrated in figure 11.

![Graph showing refractive index against wavelength](image)

Figure 11. Refractive index of sodium vapour as determined by Wood using a prism of sodium vapour.

6. **Reflection**

The reflected wave arises from the waves scattered backwards by each layer of atoms. I do not wish to discuss this phenomena in detail. However, support may be obtained for our picture of the refractive process by considering the phenomena associated with the name of Brewster. In figure 12 we have drawn a refraction-reflection situation for the case where the refracted and reflected rays are perpendicular. The incident light is polarised in the plane of scattering.

![Diagram showing Brewster's angle](image)

Figure 12. The Brewster reflection-refraction configuration.

In this configuration the atoms do not generate a reflected wave, because accelerated charges do not radiate along the line of acceleration, which in this case, coincides with the direction of the reflected wave. Thus, if the reflected wave is really generated by the atomic oscillations, in this configuration its intensity should be zero. From Snell's Law,

\[ \sin i = n \sin r \text{, but } r = \frac{\pi}{2} - i, \]

thus \[ \tan i = n. \]

This angle of incidence is known as Brewster's angle. For appropriately polarised waves the reflected intensity at Brewster's angle is experimentally zero.
7. Extensions to other Phenomena

Is it necessary that the direction of acceleration of the charges in an atom be parallel to the incident electric field? i.e. does the plane of polarisation of the scattered wave have to be parallel to that of the incident wave? The answer is no.

For most optical materials, glasses and gases, the molecules are randomly oriented and each molecule is its own mirror image. Under these conditions it is easy to show that the radiated electric field is parallel to the incident electric field. Passage of light through such materials leaves the plane of polarisation unchanged.

We consider now two cases where this is not so

(i) Optical Rotation

The sugar molecule is an example of a structure which is not its own mirror image (so are some of the molecules in turpentine). Sugar solutions rotate the plane of polarisation of light. To understand this it is simplest to analyse light scattering by a coil spring, (an example of an object which is not its own mirror image), rather than the sugar molecule itself. Consider figure 13

![Diagram of optical rotation](image)

*Figure 13.* The electromagnetic fields scattered by a coiled molecule in three different positions, a), b), c) and d). In figure a, the electric and magnetic fields of the incident wave are shown, looking along the direction of travel. Figure c shows the resultant electric and magnetic field which is slightly rotated.

When the incident field falls on the molecule oriented as in fig. 13b the electric field induces an electric dipole in the molecule. As the dipole is being set up charges spiral around the molecule from one end to the other, thereby setting up a magnetic dipole. This magnetic dipole radiates a field with the magnetic field parallel to the incident electric field. In fig. 13c the molecule has presented itself in yet another pose. In this case the incident magnetic field sets up an induced EMF in the molecule, thereby causing charges to flow from one end to the other, thereby setting up an electric dipole moment. The electric dipole radiates a wave whose electric field is parallel to the incident magnetic field. In case 13d where the molecule presents itself end on, little happens. Thus in every case where there is a scattered wave the scattered field is perpendicular to the incident field, which causes the total field to rotate as is shown in figure 13e. The overall appearance of a wave moving through such an optically active medium is shown in figure 14. Because the rate of rotation depends on the wavelength, colour phenomena observed.
(ii) Birefringence

In a crystalline material the electrons may find their motion somewhat constrained. Thus the incident electric field may find it easier to polarise the electrons in one direction than in the perpendicular direction. This results in the material having two wave-velocities i.e. two refractive indices, each belonging to a plane of polarisation perpendicular to the other.

Thus a plane polarised wave incident on a slab is resolved into two perpendicular plane polarised waves which move through the medium with different speeds. When these two fields recombine on the far side of the slab they do so with a relative phase difference which depends on the slab thickness and the two wave lengths in the medium.

When the light is examined through a polariser and the incident light was white light different thicknesses of material will give rise to different colours, depending on just which colour is most constructively produced.

A slab of varying thickness produces an exotic display in many colours, a new and challenging art form.

Apart from materials such as calcite, these phenomena may be observed in strained perspex or stretched "dulex" tape. The ability to see stress patterns in colour forms the basis of photo-elasticity.

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